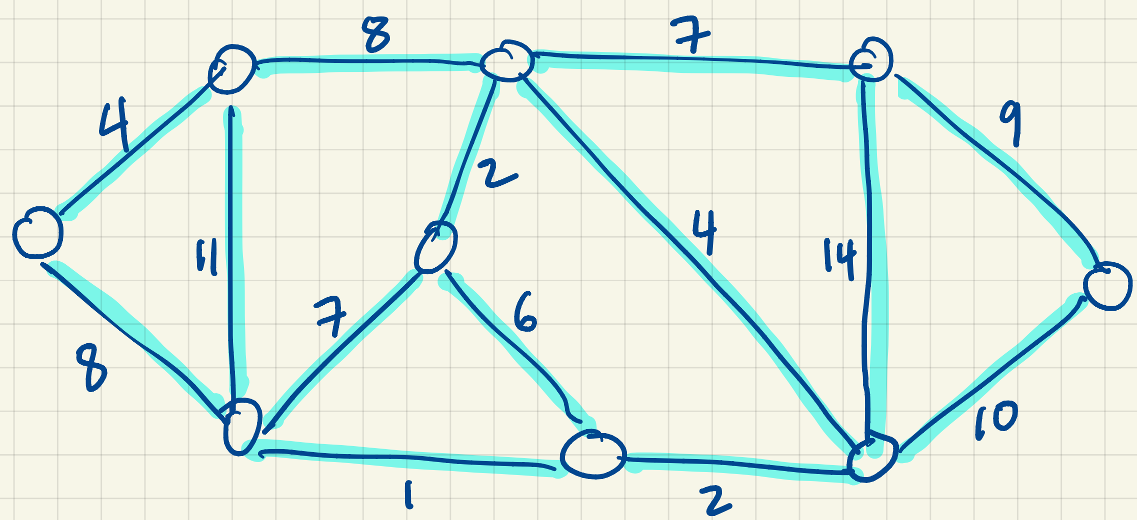
Finding the best tree : The min. Spanning tree

Given a connected weighted graph : a graph with a weight function of the edges

$$w : E \longrightarrow \mathbb{R}$$

Find a tree (connected acyclic) that has the smallest total weight.

Example :



Brute force:

Trying all possible trees is bad: There are many!

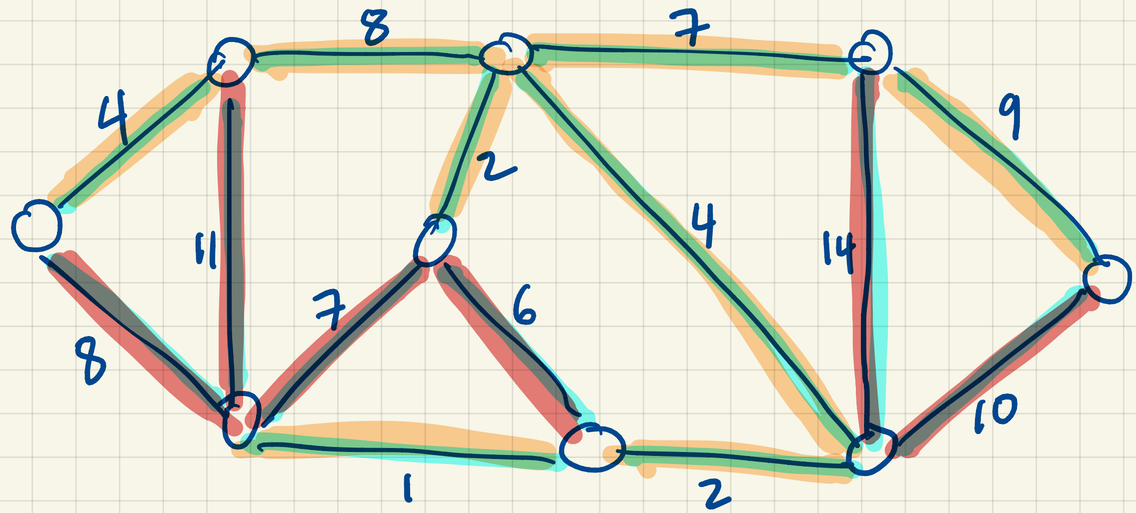
An algorithm that works: Greedy algorithm.

Pick the smallest weight edge and add it to the tree as long as it makes no cycle (so sort the edges by weight and go through them one at a time)

Remember: Sorting is $|E| \log |E|$ time.

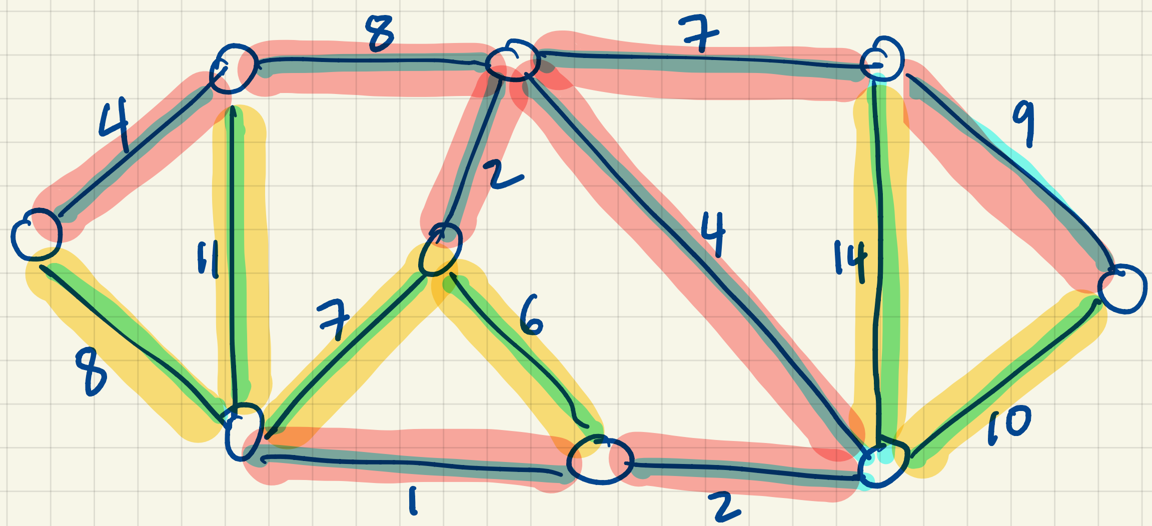
Checking for cycles can be done efficiently.

Proof that alg. produces best tree: CSCI 335/Chapter 8



8
7
4
9
11
2
4
14
7
6
8
10
1
2

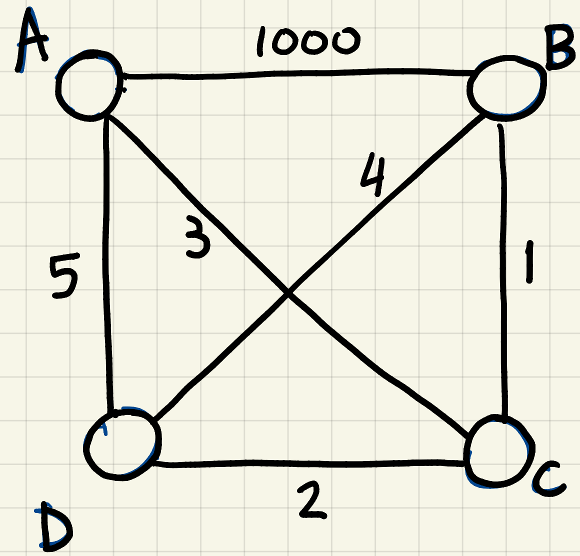
Running the greedy alg. together in class ...



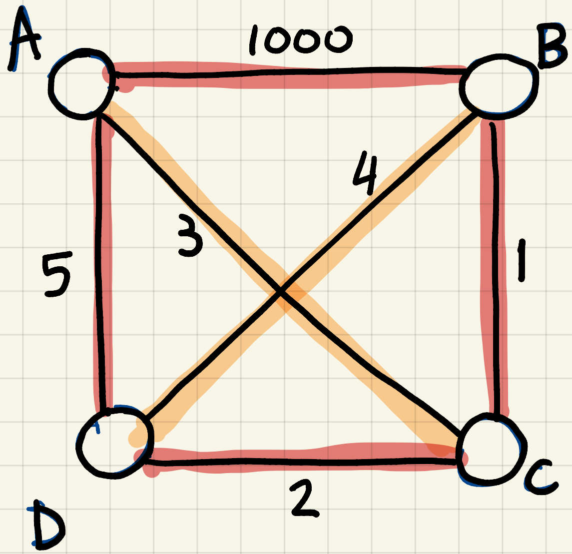
weight of tree: $1 + 2 + 2 + 4 + 4 + 7 + 8 + 9 = 37$

Greedy does not always work!

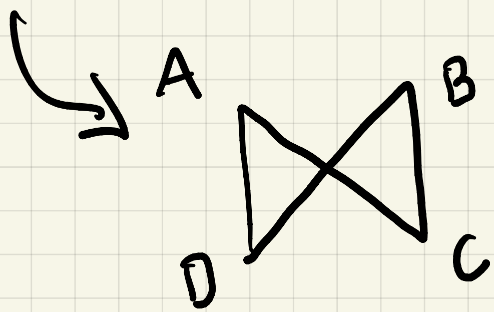
Example : Traveling salesman : Find a cycle that visits every vertex exactly once at min. cost.



Greedy : Pick the smallest weight edge and add it to cycle as long as degree of every vertex ≤ 2 and cycle does not close early (missing some)

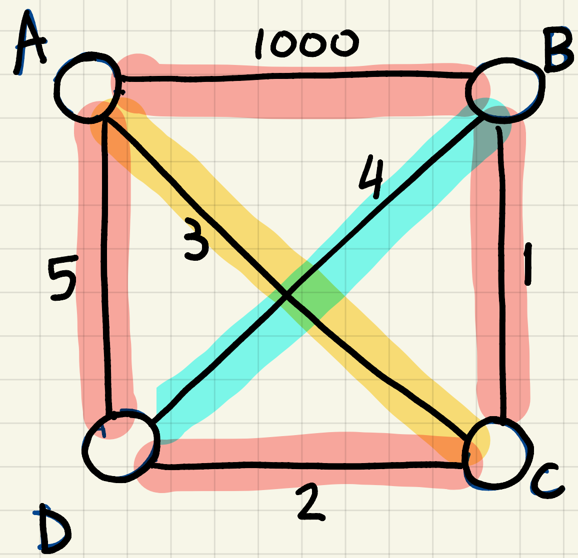


$$1+2+5+1000=1008$$



$$1+3+4+5=13$$

Running the alg. together in class ...



optimal cycle has weight $1+3+5+4 = 13$

cycle found by greedy alg. has weight $1+2+5+1000$

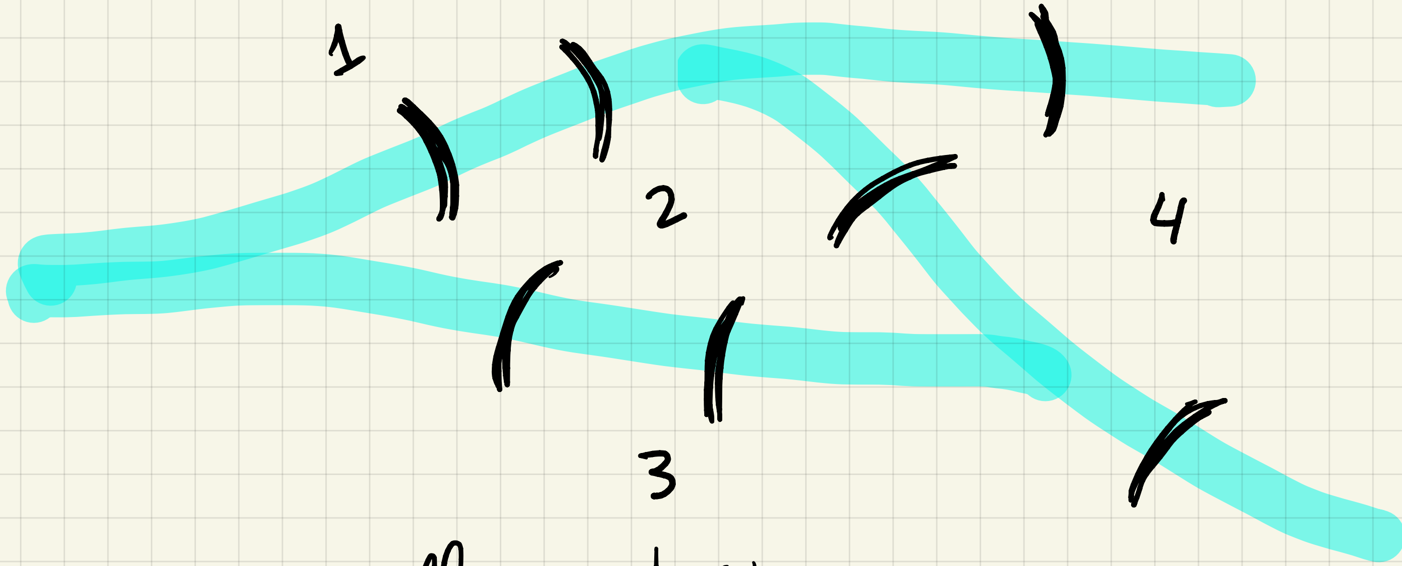
Hamiltonian Cycle: A cycle that visits every vertex exactly once.

Hard to find!

Euler cycle: A cycle that visits every edge exactly once. Easy

(Inspired by Bridges of Konigsberg, now Kaliningrad, Russia)

Euler 1735



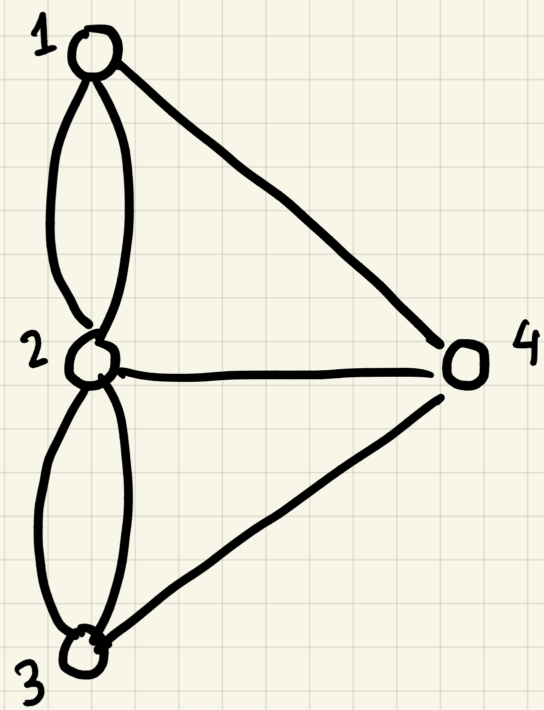
Can we cross all 7 bridges and return to where we start?

Euler's multigraph:

Graph with multiple edges between vertices

Each bridge is a edge

Find an Euler cycle.



$$\boxed{\begin{array}{c}\text{All vertices have} \\ \text{even degree}\end{array} \iff \text{Euler cycle exists.}}$$

Easy to find one:

Repeat

- Pick some arbitrary vertex
- Follow new edges arbitrarily until you can't (Found a cycle)
- Join with prev. cycle

Until done.

Example: